III Economic Growth (continued)

E Endogenous Growth: Romer's (1990) Model

1 Introduction

- Technological change (improvement in the instructions for mixing together raw materials) lies at the heart of economic growth.
- Technological change arises in large part because of *intentional* actions taken by people who respond to market incentives.
- Technology is a nonrival input. Once the cost of creating a new set of instruction has been incurred, the instructions can be used over and over again at no additional cost.

$$F(\lambda A, \lambda X) > \lambda F(A, X)$$

$$F(A, X) < A\partial F/\partial A + X\partial F/\partial X.$$

2 The Model

There are four basic inputs (physical capital, labor, human capital and technology) and three sectors (R&D, intermediate goods and final goods). The R&D sector uses human capital and the existing stock of knowledge to produce new knowledge. The intermediate goods sector uses the designs from the R&D sector together with final goods to produce producer durables. The final goods sector uses labor, human capital and producer durables to produce final output.

- Preferences:

$$\int_0^\infty e^{-\rho t} \left(\frac{c^{1-\sigma} - 1}{1 - \sigma} \right) dt. \tag{1}$$

- Final Good Production:

$$Y = H_y Y^\alpha L^\beta \int_0^A x(i)^{1-\alpha-\beta} di. \tag{2}$$

- Intermediate Good Production:

$$x(i) = K(i)/\eta. \tag{3}$$

- R&D:

$$\dot{A} = \delta H_A A. \tag{4}$$

- Market Clearing Condition (Resources Constraint):

$$\dot{K} = Y - C, \quad H = H_Y + H_A. \tag{5}$$

3 Decentralized Equilibrium

- Final Good Producers:

$$\max_{\{x(i),H_Y,L\}} \int_0^A \left[H_Y^\alpha L^\beta x(i)^{1-\alpha-\beta} - p(i)x(i)\right] di$$

$$-w_H H_Y - w_L L$$

The first-order conditions:

$$(1-\alpha-\beta)H_Y^\alpha L^\beta x(i)^{-\alpha-\beta} = p(i)$$

$$\alpha H_Y^{\alpha-1} L^\beta \int_0^A x(i)^{1-\alpha-\beta} di = w_H$$

$$\beta H_Y^\alpha L^{\beta-1} \int_0^A x(i)^{1-\alpha-\beta} di = w_L$$

- Intermediate Good Producers:

$$\pi(i) = \max_{x(i)} [p(i)x(i) - r\eta x(i)]$$

$$= \max_{x(i)} [(1-\alpha-\beta)H_Y^\alpha L^\beta x(i)^{1-\alpha-\beta} - r\eta x(i)]$$

The first-order condition:

$$(1-\alpha-\beta)^2 H_Y^\alpha L^\beta x(i)^{-\alpha-\beta} = r\eta,$$

which implies:

$$p(i) = \bar{p} \equiv \frac{r\eta}{1-\alpha-\beta}$$

$$x(i) = \bar{x} \equiv \left[\frac{(1-\alpha-\beta)^2 H_Y^\alpha L^\beta}{r\eta}\right]^{\frac{1}{\alpha+\beta}}$$

$$\pi(i) = \pi \equiv \bar{p}\bar{x} - r\eta\bar{x} = (\alpha+\beta)\bar{p}\bar{x}$$

$$= (\alpha+\beta)(1-\alpha-\beta)H_Y^\alpha L^\beta \bar{x}^{1-\alpha-\beta}.$$

- R&D Firms:

$$P_A(t) = \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds}\pi(\tau)d\tau$$

$$\dot{P}_A(t) = \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds}\pi(\tau)r(t)d\tau - e^{-\int_t^{t} r(s)ds}\pi(t)$$

$$= r(t)P_A(t) - \pi(t),$$

which implies:

$$r(t)P_A = \pi(t).$$

- Consumers:

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\sigma}.$$

- The Solution (Balanced Growth Equilibrium): The variables A, K, Y and C grow at constant rates.

$$P_A = \frac{\pi}{r} = \frac{(\alpha+\beta)\bar{p}\bar{x}}{r} = \left(\frac{\alpha+\beta}{r}\right)(1-\alpha-\beta)H_Y^{\alpha}L^{\beta}\bar{x}^{1-\alpha-\beta}$$

Human capital H is allocated in a way such that

$$w_H = P_A\delta A = \alpha H_Y^{\alpha-1}L^{\beta}A\bar{x}^{1-\alpha-\beta}$$

$\Rightarrow$

$$H_Y = \frac{\alpha r}{\delta(\alpha+\beta)(1-\alpha-\beta)}$$

$$H_A = H - \Lambda r/\delta, \quad \Lambda = \frac{\alpha}{(\alpha+\beta)(1-\alpha-\beta)}$$

From

$$Y = H_Y^{\alpha}L^{\beta}A\bar{x}^{1-\alpha-\beta}$$

we have $\dot{Y}/Y = \dot{A}/A$ if L and $\bar{x}$ are fixed.

From $K = A\bar{x}\eta$, we get $\dot{K}/K = \dot{A}/A$ if $\bar{x}$ is fixed.

From $C = Y - \dot{K}$, we obtain $C/Y = 1 - \dot{K}/Y = 1 - (\dot{K}/K)(K/Y)$, which, along with the fact that K/Y is constant, implies $\dot{C}/C = \dot{Y}/Y$. So we have

$$g = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \delta H_A = \delta H - \Lambda r. \tag{6}$$

In addition, we have

$$g = \frac{r - \rho}{\sigma}. \tag{7}$$

Solving (6) and (7) gives the decentralized equilibrium growth rate:

$$g = \frac{\delta H - \Lambda\rho}{1 + \sigma\Lambda}.$$

4 Social Planner's Solution

- The social planner chooses C and H_A to maximize (1) subject to the following constraints:

$$\dot{K} = \eta^{\alpha+\beta-1} A^{\alpha+\beta} (H - H_A)^{\alpha} L^{\beta} K^{1-\alpha-\beta} - C \equiv \Delta - C,$$

and

$$\dot{A} = \delta H_A A.$$

- The current-value Hamiltonian:

$$\mathcal{H} = \frac{c^{1-\sigma} - 1}{1 - \sigma} + \lambda(\Delta - C) + \mu\delta H_A A.$$

The first-order conditions:

$$C^{-\sigma} - \lambda = 0, \tag{8}$$

$$\lambda\alpha\Delta/(H - H_A) + \mu\delta A = 0 \tag{9}$$

$$\lambda(1-\alpha-\beta)\Delta/K = \rho\lambda - \dot{\lambda}, \tag{10}$$

$$\lambda(\alpha+\beta)\Delta/A + \mu\delta H_A = \rho\mu - \dot{\mu}, \tag{11}$$

$$\dot{K} = \Delta - C, \tag{12}$$

$$\dot{A} = \delta H_A A, \tag{13}$$

and TVCs.

$$(8) \Rightarrow \frac{\dot{\lambda}}{\lambda} = -\sigma\frac{\dot{c}}{c} = -\sigma\frac{\dot{A}}{A} = -\sigma\delta H_A \tag{14}$$

$$(9) \Rightarrow \frac{\lambda}{\mu} = \frac{\delta A(H - H_A)}{\alpha\Delta} \tag{15}$$

$$(11) \Rightarrow \frac{\dot{\mu}}{\mu} = \rho - \delta H_A - \frac{\lambda}{\mu}\frac{(\alpha+\beta)\Delta}{A}$$

$$= \rho - \delta H_A - \frac{\delta(\alpha+\beta)(H - H_A)}{\alpha}$$

Setting $\dot{\mu}/\mu = \dot{\lambda}/\lambda$ gives:

$$-\sigma\delta H_A = \rho - \frac{\delta(\alpha+\beta)}{\alpha}H - \rho$$

which implies:

$$\delta H_A = \frac{\delta H - \Theta\rho}{\Theta\sigma + (1-\Theta)}, \qquad \Theta = \frac{\alpha}{\alpha+\beta}$$

$$g^* = \frac{\delta H - \Theta\rho}{\Theta\sigma + (1-\Theta)}.$$

- Decentralized equilibrium is not Pareto optimal ($g < g^*$): (i). Monopoly pricing: Λ vs. Θ. (ii). External effects of producing new ideas: 1 vs. $1 - \Theta$.